Table 8.5.1. Empirical cumulative distribution of $n(\hat{\rho}-1)$ for $\rho = 1$

Sample Size	Probability of a Smaller Value							
n	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
				$\hat{\rho}$				
25	−11.9	−9.3	−7.3	−5.3	1.01	1.40	1.79	2.28
50	−12.9	−9.9	−7.7	−5.5	0.97	1.35	1.70	2.16
100	−13.3	−10.2	−7.9	−5.6	0.95	1.31	1.65	2.09
250	−13.6	−10.3	−8.0	−5.7	0.93	1.28	1.62	2.04
500	−13.7	−10.4	−8.0	−5.7	0.93	1.28	1.61	2.04
∞	−13.8	−10.5	−8.1	−5.7	0.93	1.28	1.60	2.03
				$\hat{\rho}_\mu$				
25	−17.2	−14.6	−12.5	−10.2	−0.76	0.01	0.65	1.40
50	−18.9	−15.7	−13.3	−10.7	−0.81	−0.07	0.53	1.22
100	−19.8	−16.3	−13.7	−11.0	−0.83	−0.10	0.47	1.14
250	−20.3	−16.6	−14.0	−11.2	−0.84	−0.12	0.43	1.09
500	−20.5	−16.8	−14.0	−11.2	−0.84	−0.13	0.42	1.06
∞	−20.7	−16.9	−14.1	−11.3	−0.85	−0.13	0.41	1.04
				$\hat{\rho}_\tau$				
25	−22.5	−19.9	−17.9	−15.6	−3.66	−2.51	−1.53	−0.43
50	−25.7	−22.4	−19.8	−16.8	−3.71	−2.60	−1.66	−0.65
100	−27.4	−23.6	−20.7	−17.5	−3.74	−2.62	−1.73	−0.75
250	−28.4	−24.4	−21.3	−18.0	−3.75	−2.64	−1.78	−0.82
500	−28.9	−24.8	−21.5	−18.1	−3.76	−2.65	−1.78	−0.84
∞	−29.5	−25.1	−21.8	−18.3	−3.77	−2.66	−1.79	−0.87

NOTE. This table was constructed by David A. Dickey using the Monte Carlo method. Details are given in Dickey (1975). Standard errors of the estimates vary, but most are less than 0.15 for entries in the left half of the table and less than 0.03 for entries in the right half of the table.

Although the sign of e_{t-j} in the weighted sum $\sum_{j=0}^{t-1}(-1)^j e_{t-j}$ is not the same for all t, the sign is always opposite of that for e_{t-j-1} and e_{t-j+1}, and it follows that

$$\sum_{t=1}^{n-1} X_t^2 = \sum_{t=1}^{n-1}\left(\sum_{j=1}^{t}(-1)^j e_j\right)^2.$$

The distribution of e_t, $t = 1, 2, \ldots$, is symmetric and hence the distributional properties of the sequence $-e_1, e_2, -e_3, e_4, \ldots$, are precisely the same as the

Table 8.5.2. Empirical cumulative distribution of $\hat{\tau}$ for $\rho=1$

Sample Size	Probability of a Smaller Value							
n	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
				$\hat{\tau}$				
25	−2.66	−2.26	−1.95	−1.60	0.92	1.33	1.70	2.16
50	−2.62	−2.25	−1.95	−1.61	0.91	1.31	1.66	2.08
100	−2.60	−2.24	−1.95	−1.61	0.90	1.29	1.64	2.03
250	−2.58	−2.23	−1.95	−1.62	0.89	1.29	1.63	2.01
500	−2.58	−2.23	−1.95	−1.62	0.89	1.28	1.62	2.00
∞	−2.58	−2.23	−1.95	−1.62	0.89	1.28	1.62	2.00
				$\hat{\tau}_{\mu}$				
25	−3.75	−3.33	−3.00	−2.63	−0.37	0.00	0.34	0.72
50	−3.58	−3.22	−2.93	−2.60	−0.40	−0.03	0.29	0.66
100	−3.51	−3.17	−2.89	−2.58	−0.42	−0.05	0.26	0.63
250	−3.46	−3.14	−2.88	−2.57	−0.42	−0.06	0.24	0.62
500	−3.44	−3.13	−2.87	−2.57	−0.43	−0.07	0.24	0.61
∞	−3.43	−3.12	−2.86	−2.57	−0.44	−0.07	0.23	0.60
				$\hat{\tau}_{\tau}$				
25	−4.38	−3.95	−3.60	−3.24	−1.14	−0.80	−0.50	−0.15
50	−4.15	−3.80	−3.50	−3.18	−1.19	−0.87	−0.58	−0.24
100	−4.04	−3.73	−3.45	−3.15	−1.22	−0.90	−0.62	−0.28
250	−3.99	−3.69	−3.43	−3.13	−1.23	−0.92	−0.64	−0.31
500	−3.98	−3.68	−3.42	−3.13	−1.24	−0.93	−0.65	−0.32
∞	−3.96	−3.66	−3.41	−3.12	−1.25	−0.94	−0.66	−0.33

This table was constructed by David A. Dickey using the Monte Carlo method. Details are given in Dickey (1975). Standard errors of the estimates vary, but most are less than 0.02.

To extend the results for the first order process with $\rho=1$ to the pth order autoregressive process, we consider the time series

$$Y_t = \sum_{j=1}^{t} Z_j, \qquad t = 1, 2, \ldots, \tag{8.5.11}$$

where $\{Z_t \colon t \in (0, \pm 1, \pm 2, \ldots)\}$ is a $(p-1)$ order autoregressive time series with the representation

$$Z_t + \sum_{i=2}^{p} a_i Z_{t-i+1} = e_t, \tag{8.5.12}$$

TABLE B.7
Critical Values for the Dickey-Fuller Test Based on the *OLS F* Statistic

Sample size T	*Probability that F test is greater than entry*							
	0.99	*0.975*	*0.95*	*0.90*	*0.10*	*0.05*	*0.025*	*0.01*
				Case 2				
	(F test of $\alpha = 0$, $\rho = 1$ in regression $y_t = \alpha + \rho y_{t-1} + u_t$)							
25	0.29	0.38	0.49	0.65	4.12	5.18	6.30	7.88
50	0.29	0.39	0.50	0.66	3.94	4.86	5.80	7.06
100	0.29	0.39	0.50	0.67	3.86	4.71	5.57	6.70
250	0.30	0.39	0.51	0.67	3.81	4.63	5.45	6.52
500	0.30	0.39	0.51	0.67	3.79	4.61	5.41	6.47
∞	0.30	0.40	0.51	0.67	3.78	4.59	5.38	6.43
				Case 4				
	(F test of $\delta = 0$, $\rho = 1$ in regression $y_t = \alpha + \delta t + \rho y_{t-1} + u_t$)							
25	0.74	0.90	1.08	1.33	5.91	7.24	8.65	10.61
50	0.76	0.93	1.11	1.37	5.61	6.73	7.81	9.31
100	0.76	0.94	1.12	1.38	5.47	6.49	7.44	8.73
250	0.76	0.94	1.13	1.39	5.39	6.34	7.25	8.43
500	0.76	0.94	1.13	1.39	5.36	6.30	7.20	8.34
∞	0.77	0.94	1.13	1.39	5.34	6.25	7.16	8.27

The probability shown at the head of the column is the area in the right-hand tail.

Source: David A. Dickey and Wayne A. Fuller, "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root," *Econometrica* 49 (1981), p. 1063.

TABLE B.7
Critical Values for the Dickey-Fuller Test Based on the *OLS F* Statistic

Sample size *T*	*Probability that F test is greater than entry*							
	0.99	*0.975*	*0.95*	*0.90*	*0.10*	*0.05*	*0.025*	*0.01*
				Case 2				
	(F test of $\alpha = 0$, $\rho = 1$ in regression $y_t = \alpha + \rho y_{t-1} + u_t$)							
25	0.29	0.38	0.49	0.65	4.12	5.18	6.30	7.88
50	0.29	0.39	0.50	0.66	3.94	4.86	5.80	7.06
100	0.29	0.39	0.50	0.67	3.86	4.71	5.57	6.70
250	0.30	0.39	0.51	0.67	3.81	4.63	5.45	6.52
500	0.30	0.39	0.51	0.67	3.79	4.61	5.41	6.47
∞	0.30	0.40	0.51	0.67	3.78	4.59	5.38	6.43
				Case 4				
	(F test of $\delta = 0$, $\rho = 1$ in regression $y_t = \alpha + \delta t + \rho y_{t-1} + u_t$)							
25	0.74	0.90	1.08	1.33	5.91	7.24	8.65	10.61
50	0.76	0.93	1.11	1.37	5.61	6.73	7.81	9.31
100	0.76	0.94	1.12	1.38	5.47	6.49	7.44	8.73
250	0.76	0.94	1.13	1.39	5.39	6.34	7.25	8.43
500	0.76	0.94	1.13	1.39	5.36	6.30	7.20	8.34
∞	0.77	0.94	1.13	1.39	5.34	6.25	7.16	8.27

The probability shown at the head of the column is the area in the right-hand tail.

Source: David A. Dickey and Wayne A. Fuller, "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root," *Econometrica* 49 (1981), p. 1063.