

McGill University
ECON 763
Financial econometrics
Mid-term exam

No documentation allowed
Time allowed: 1.5 hour

25 points 1. Answer by TRUE, FALSE or UNCERTAIN to each one of the following statements. Justify briefly your answer. (Maximum: one page per question.)

- (a) If a random variable has finite second moments, it has finite moments at all higher orders.
- (b) Any stationary process of order 5 is also stationary of order 2.
- (c) Any strictly stationary process is in L_2 .
- (d) The Wold theorem holds for finite-order moving average processes but not autoregressive processes.
- (e) Non-invertible moving processes have no covariance generating function.

50 points 2. Consider the following models:

$$X_t = 10 + 0.7 X_{t-1} - 0.2 X_{t-2} + u_t \quad (1)$$

where $\{u_t : t \in \mathbb{Z}\}$ is an *i.i.d.* $N(0, 1)$ sequence. For each one of these models, answer the following questions.

- (a) Is this model stationary? Why?
- (b) Is this model invertible? Why?
- (c) Compute:
 - i. $E(X_t)$;
 - ii. $\gamma(k)$, $k = 1, \dots, 8$;
 - iii. $\rho(k)$, $k = 1, 2, \dots, 8$.

- (d) Graph $\rho(k)$, $k = 1, 2, \dots, 8$.
- (e) Find the coefficients of $u_t, u_{t-1}, u_{t-2}, u_{t-3}$ and u_{t-4} in the moving average representation of X_t .
- (f) Find the autocovariance generating function of X_t .
- (g) Compute the first two partial autocorrelations of X_t .
- (h) If $X_{10} = 11$, compute the best linear forecast of X_{11} based on X_{10} (only). Justify your answer.
- (i) If $X_8 = 12, X_9 = 9$ and $X_{10} = 11$, compute the best linear forecast of X_{11} and X_{12} based on the past X_t up to time 10. Justify your answer.

25 points 3. Let X_1, X_2, \dots, X_T be a time series.

- (a) Define:
 - i. the sample autocorrelations for this series;
 - ii. the partial autocorrelations for this series.
- (b) Discuss the asymptotic distributions of these two sets of autocorrelations in the following cases:
 - i. under the hypothesis that X_1, X_2, \dots, X_T are independent and identically distributed (i.i.d.);
 - ii. under the hypothesis that the process follows a moving average of finite order.
- (c) Propose a method for testing the hypothesis that X_1, X_2, \dots, X_T are independent and identically distributed (i.i.d.) without any assumption on the existence of moments for X_1, X_2, \dots, X_T .