

McGill University
ECON 763
Financial econometrics
Mid-term exam

No documentation allowed
Time allowed: 1.5 hour

- 30 points 1. Answer by TRUE, FALSE or UNCERTAIN to each one of the following statements. Justify briefly your answer. (Maximum: one page per question.)
- (a) If a random variable has finite second moments, it has finite moments at all higher orders.
 - (b) Any stationary process of order 5 is also stationary of order 2.
 - (c) Any strictly stationary process is in L_2 .
 - (d) The Wold theorem holds for finite-order moving average processes but not autoregressive processes.
 - (e) Non-invertible moving processes have no covariance generating function.
- 20 points 2. Let $\gamma(k)$ the autocovariance function of second-order stationary process on the integers. Prove that:
- (a) $\gamma(0) = \text{Var}(X_t)$ et $\gamma(k) = \gamma(-k)$, $\forall k \in \mathbb{Z}$;
 - (b) $|\gamma(k)| \leq \gamma(0)$, $\forall k \in \mathbb{Z}$;
 - (c) the function $\gamma(k)$ is positive semi-definite.
- 50 points 3. Consider the following models:

$$X_t = 10 + u_t - 0.75 u_{t-1} + 0.125 u_{t-2}, \quad (1)$$

where $\{u_t : t \in \mathbb{Z}\}$ is an *i.i.d.* $N(0, 1)$ sequence. For each one of these models, answer the following questions.

- (a) Is this model stationary? Why?
- (b) Is this model invertible? Why?
- (c) Compute:
 - i. $E(X_t)$;
 - ii. $\gamma(k)$, $k = 1, \dots, 8$;
 - iii. $\rho(k)$, $k = 1, 2, \dots, 8$.
- (d) Graph $\rho(k)$, $k = 1, 2, \dots, 8$.
- (e) Find the coefficients of $u_t, u_{t-1}, u_{t-2}, u_{t-3}$ and u_{t-4} in the moving average representation of X_t .
- (f) Find the autocovariance generating function of X_t .
- (g) Find and graph the spectral density of X_t .
- (h) Compute the first two partial autocorrelations of X_t .